U3A MANNINGHAM UNIVERSITY OF THE THIRD AGE

U3AM at Home

MUCH ADO ABOUT NOTHING – PART 2

The Importance of Zero

Last week we looked at the early numbering systems and the clumsiness of the way in which they were expressed. As mentioned, most numbering systems were based on 10, because of the human anatomy. That could equally have developed into a base 20, for those who did not wear socks. In fact, the Aztecs of Central America and the Muisca of South America both used a base 20 system, based on counting fingers and toes. The Muisca had discrete numerals from 1 to 10, then used “foot” 1 to “foot” 9 to represent eleven to nineteen. Twenty was regarded as the “golden number” and higher numbers were expressed in terms of twenty, so seventy was three times twenty plus ten.

The Aztecs used dots to represent numbers up to nineteen, then a flag for twenty, repeating this as many times as necessary up to four hundred (twenty times twenty), which was represented by a sign depicting a fir tree (meaning as numerous as hairs), then the next unit, for eight thousand (twenty times twenty times twenty) was represented by an incense bag, which referred to the almost uncountable contents of a sack of cacao beans.

The ancient Babylonians used a base of 60. They in turn adopted this from the ancient Sumerians. From this we get a number of our measuring units – 60 seconds in a minute, 60 minutes in an hour. (The number of hours in a day came from the ancient Egyptians who split daylight into 10 hours together with a twilight hour at each end, then the same divisions for the night, giving 24 hours in total.

It was not until the third century AD that the idea of “place” was born, somewhere in northern India. This meant that the only characters needed to represent a number were those which make up the characters of the base of the numbering system – in the decimal system 1 to 9.

In the decimal system each place in a number representation counting from the right is 10 times the value of each “digit” in the next position to the right. Thus, fifteen can be written as 15 meaning one times ten plus five. Two hundred and seventy-three can be written as 273, meaning 2 times one hundred (ten times ten) plus 7 times ten plus three. With this system, we can represent almost any number with a string of digits from 1 to 9. I say almost because this does not take into account places which do not have a value of 1 to 9 in any of the places, such as one hundred and two, three thousand and forty-five. Obviously twelve (12) must be distinguished from one hundred and two (102) and three hundred and forty-five (345) must be distinguished from three thousand and forty five (3045).

Believe it or not, it took another one hundred years to realise the need for a character to represent no value in a place – zero. We can then write the first of these as 102 and the second as 3045.

This seems to be so obvious to us with the benefit of hundreds of years of study behind what we learn – but remember that it took thousands of years to arrive at the place system so in the light of that, another hundred years is quite short. So now one hundred and two is 1 times ten times ten 0 times 10 + 2 + 0 times ten. To simplify the writing of these numbers, we use x to specify times, + to specify plus and we use a superscript of 10 to indicate how many 10s to multiply for each place. So 102 is now $1x10^2 + 0x10^1 + 2x10^0$, (1x10x10+0x10+2) and 3045 is $3x10^3 + 0x10^2 + 4x10^1 + 5x10^0$, (3x10x10x10+0x10x10+4x10+5). There is nothing mysterious about using superscripts, they are just a shorthand for telling how many 10s are to be multiplied in that place, just as we use x instead of times and + instead of plus.

0 and 1 are common to all modern numbering systems: binary (base 2) "digits" 0 and 1, octal (base 8) "digits" 0 to 7, decimal (base 10) "digits" 0 to 9, and hexadecimal (base 16) 0 to 15 where 11 to 15 are represented by A to F (because each place must be represented by just one character).

The binary system (the basis of all electronics) is easy to represent by two states – on and off or plus and minus or 0 and 1. 1 is 1, 2 is 10, 3 is 11, 4 is 100, 5 is 101, 6 is 110, 7 is 111, etc.

While working in binary is very useful because the two states are easy to produce a real-world circuit, the numbers start to get unwieldy as they grow. For example, 64 in binary is 100000 and the size escalates from there. That is why, in computer addressing, we use hexadecimal (base 16) so that the number of places does not grow so quickly.

Warwick Wright