Arabic and Persian Mathematics (Part 3)

In reading about these areas of mathematics that we often take as obvious. it often was, once someone had worked it out beforehand. This series is about some of the lesser-known of those people.

Figure 1 A page from al-Khwārizmī's Algebra

This book is considered the foundational text of modern algebra. It provided an exhaustive account of solving polynomial equations up to the second degree, and introduced the fundamental methods of "reduction" and "balancing", referring to the transposition of subtracted terms to the other side of an equation, that is, the cancellation of terms on opposite sides of the equation.

Al-Khwārizmī's method of solving linear and quadratic equations worked by first reducing the equation to one of six standard forms (where b and c are positive integers)

* squares equal roots ($ax^2 = bx$)

* squares equal number ($ax^2 = c$)

* roots equal number ($bx = c$)

* squares and roots equal number ($ax^2 + bx = c$)

* squares and number equal roots ($ax^2 + c = bx$)

* roots and number equal squares ($bx + c = ax^2$)

by dividing out the coefficient of the square and using the two operations al-jabr (“restoring” or “completion”) and al-muqābala ("balancing"). Al-jabr is the process of removing negative units, roots and squares from the equation by adding the same quantity to each side. For example, $x^2 = 40x - 4x^2$ is reduced to $5x^2 = 40x$. Al-muqābala is the process of bringing quantities of the same type to the same side of the equation. For example, $x^2 + 14 = x + 5$ is reduced to $x^2 + 9 = x$.

The above discussion uses modern mathematical notation for the types of problems which the book discusses. However, in al-Khwārizmī's day, most of this notation had not yet been invented, so he had to use ordinary text to present problems and their solutions. For example, for one problem he writes, (from an 1831 translation)

"If someone says: "You divide ten into two parts: multiply the one by itself; it will be equal to the other taken eighty-one times." Computation: You say, ten less thing, multiplied by itself, is a hundred plus a square less twenty things, and this is equal to eighty-one things. Separate the twenty things from a hundred and a square, and add them to eighty-one. It will then be a hundred plus a square, which is equal to a hundred and one roots. Halve the roots; the moiety is fifty and a half. Multiply this by itself, it is two thousand five hundred and fifty and a quarter. Subtract from this one hundred; the remainder is two thousand four hundred and fifty and a quarter. Extract the root from this; it is forty-nine and a half. Subtract this from the moiety of the roots, which is fifty and a half. There remains one, and this is one of the two parts."

In modern notation this process, with 'x' the "thing" (shay') or "root", is given by the steps:

$(10-x)^2$

$(=100+x^2-20x)=81x$

$x^2+100=101x$

Let the roots of the equation be ‘p’ and ‘q’. Then (p+q)/2 =50½, pq=100 and
$(p-q)/2=\mathrm{Sqrt}(((p+q)/2)^2-pq)$
=Sqrt(2550¼-100)=49 ½

So a root is given by
X=50 ½-49½=1
Al-Khwarizmi's successors undertook a systematic application of arithmetic to algebra, algebra to arithmetic, both to trigonometry, algebra to the Euclidean theory of numbers, algebra to geometry, and geometry to algebra. This was how the creation of polynomial algebra, combinatorial analysis, numerical analysis, the numerical solution of equations, the new elementary theory of numbers, and the geometric construction of equations arose.

Next time we will look at, not an individual, but a civilisation whose importance was lost to the old world by a simple matter of geography.