Indian Mathematics

In reading about these areas of mathematics that we often take as obvious. it often was, once someone had worked it out beforehand. This series is about some of the lesser-known of those people.

Place value system and zero

The place-value system, first seen in the Bakhshali Manuscript, which has been dated between 200BC and 300AD, was clearly in place in his work. The author of the Bakhshali Manuscript is not known.

While Aryabhata did not use a symbol for zero, the French mathematician Georges Ifrah argues that knowledge of zero was implicit in Aryabhata's place-value system as a place holder for the powers of ten with null coefficients.

However, Aryabhata did not use the brahmi numerals. Continuing the Sanskritic tradition from Vedic times, he used letters of the alphabet to denote numbers, expressing quantities, such as the table of sines in a mnemonic form.

Pi as irrational

Aryabhata worked on the approximation for Pi (π), and may have come to the conclusion that π is irrational. In the second part of the Aryabhatiyam he writes:

"Add four to 100, multiply by eight, and then add 62,000. By this rule the circumference of a circle with a diameter of 20,000 can be approached."

This implies that the ratio of the circumference to the diameter is ((4+100)×8+62000)/20000 = 3.1416, which is accurate to five significant figures.

It is speculated that Aryabhata used the term "approaching" to mean that not only is this an approximation but that the value is incommensurable (or irrational). If this is correct, it is quite a sophisticated insight, because the irrationality of pi was proved in Europe only in 1761 by Lambert).

After Aryabhatiya was translated into Arabic (ca. 820 AD) this approximation was mentioned in Al-Khwarizmi's book on algebra.

Algebra

In Aryabhatiya, Aryabhata provided elegant results for the summation of series of squares and cubes:

$$1^2 + 2^2 + \ldots + n^2 = \frac{(n + 1)(2n + 1)}{6}$$

and

$$1^3 + 2^3 + \ldots + n^3 = (1+2+ \ldots +n)^2$$

Aryabhata's work influenced the Indian astronomical tradition and several neighbouring cultures through translations. His definition of sine, cosine and several other trigonometrical functions influenced the birth of trigonometry. He was also the first to specify sine and versine (1-cos x) tables in 3.75° intervals to 90° to an accuracy of 4 decimal places.

Aryabhata's astronomical calculation methods were also very influential. Along with the trigonometric tables, they came to be widely used in the Islamic world and used to compute many Arabic astronomical tables (zijes). In particular, the astronomical tables in the work of the Arabic Spain scientist Al-Zarqali (11^{th} century) were translated into Latin as the Tables of Toledo (12^{th} century) and remained the most accurate ephemeris used in Europe for centuries.

Calendric calculations devised by Aryabhata and his followers have been in continuous use in India for the practical purposes of fixing the Panchangam (the Hindu calendar). In the Islamic world, they formed the basis of the Jalali calendar introduced in 1073 AD by a group of astronomers including Omar Khayyam, versions of which (modified in 1925) are the national calendars in use in Iran and Afghanistan today. The dates of the Jalali calendar are based on actual solar transit, as in Aryabhata and earlier Siddhanta calendars. This type of calendar requires an ephemeris for calculating dates. Although dates were difficult to compute, seasonal errors were less in the Jalali calendar than in the Gregorian calendar.

India's first satellite Aryabhata and the lunar crater Aryabhata are named in his honour. An Institute for conducting research in astronomy, astrophysics and atmospheric sciences is the Aryabhatta Research Institute of observational sciences (ARIES) near Nainital, India. The inter-school Aryabhata Maths Competition is also named after him, as is Bacillus aryabhata, a species of bacteria discovered by ISRO scientists in 2009.

In 458 AD, Indian mathematicians wrote a book, the Lokavibhaaga, that uses zero in this way. In 628 AD, Brahmagupta wrote a book explaining how zero worked, with rules like "The sum of zero and zero is zero" and "The sum of a positive and a negative is their difference; or, if they are equal, zero.". With the formation of the Islamic Empire a few years later, the use of zero spread quickly from India to West Asia and Africa (by the 800's), and then more slowly to Christian Europe (not until the 1200's AD, and only specialists used it until the 1500's).

A later landmark in Indian mathematics was the development of the series expansions for trigonometric functions (sine, cosine, and arc tangent) by mathematicians of the Kerala School in the fifteenth century AD. Their remarkable work, completed two centuries before the invention of calculus in Europe, provided what is now considered the first example of a power series (apart from the geometric series). However, they did not formulate a systematic theory of differentiation and integration, nor is there any direct evidence of their results being transmitted outside Kerala.

Baudhayana (c. 8th century BCE) composed the Baudhayana Sulba Sutra, the best-known Sulba Sutra, which contains examples of simple Pythagorean triples, such as: (3,4,5), (5,12,13), (8,15,17), (7,24,25), and (12,35,37) as well as a statement of the Pythagorean theorem for the sides of a square. Baudhayana also gave a formula for the square root of two giving 1.4142156….

The formula is accurate up to five decimal places, the true value being 1.41421356...