Historical Figures in Mathematics

In reading about these areas of mathematics that we often take as obvious. it often was, once someone had worked it out beforehand. This series is about some of the lesser-known of those people.

Zu Chongzhi: the calculation of PI (π) (conclusion)

Last time we met Chinese mathematician and polymath Zu Chonzhi and his much more accurate calendar whose adoption was opposed by an influential minister of the Emperor

Zu replied that his calendar was:-

... not from spirits or from ghosts, but from careful observations and accurate mathematical calculations. ... people must be willing to hear and look at proofs in order to understand truth and facts.

Despite having such a powerful opponent as Tai Faxin, Zu won approval for his calendar from Emperor Xiao-wu and the Tam-ing calendar was due to come into use in 464. However, Xiao-wu died in 464 before the calendar was introduced, and his successor was persuaded by Tai Faxin to cancel the introduction of the new calendar. Zu left the imperial service on the death of Emperor Xiao-wu and devoted himself entirely to his scientific studies.

Of course, it is not unreasonable to ask where the numbers 144 and 391 came from. Having accurate knowledge of the lengths of the year and the month were necessary, but it is still not clear how Zu translated this into a cycle of 391 years. It has been suggested that Zu found that there were 365 $^{9589}/_{39491}$ days in a year and $^{116321}/_{3939}$ days in a month. This gives

$12^{1691772624}/_{4593632611}$

months in a year. But Zu would know how to reduce fractions to their lowest terms by dividing top and bottom by the greatest common divisor. Doing this gives

$^{1691772624}/_{4593632611} = {}^{144}/_{391}$

(the common devisor being 11748421, of course)
and hence the extra month in 144 out of 391 years.

Before we leave our discussion of Zu's astronomical work we give further details of his work in this area. He was not the first Chinese astronomer to discover the precession of the equinoxes (Yu Xi did so in the fourth century) but he was the first to take this into account in calendar calculations. Because of the precession of the equinoxes the tropical year is shorter by about 21 minutes than the sidereal year (the time taken by the Sun to return to the same place against the background stars). Zu's calculations of the length of the year were well within the range that allowed him to differentiate between the tropical and sidereal year. Jupiter takes about 12 years to complete its orbit but Zu was able to give a much more accurate value

than that. He discovered that in 7 cycles of 12 years, Jupiter had completed seven and one twelfth orbits, giving its sidereal period as 11.859 years (accurate to within one part in 4000).

He gave the rational approximation $^{355}/_{113}$ to π in his text *Zhui shu* (Method of Interpolation), which is correct to 6 decimal places. He also proved that

$$3.1415926 < \pi < 3.1415927$$

a remarkable result about which it would be nice to have more details. Sadly Zu Chongzhi's book is lost. It is reported in the *History of the Sui dynasty*, compiled in the 7th century by Li Chunfeng and others, that:-

Zu Chongzhi further devised a precise method [of calculating]. Taking a circle of diameter 10,000,000 chang, he found the circumference of this circle to be less than 31,415,927 chang and greater than 31,415,926 chang. He deduced from these results that the accurate value of the circumference must lie between these two values. Therefore the precise value of the ratio of the circumference of a circle to its diameter is as 355 to 113, and the approximate value is as 22 to 7.

To compute this accuracy for π, Zu must have used an inscribed regular polygon of 24,576 sides and undertaken the extremely lengthy calculations, involving hundreds of square roots, all to 9 decimal place accuracy. Since his book is lost we will never know exactly how he found the rational approximation $^{355}/_{113}$ from the decimal approximation. Historians believe, however, that he knew that

If $^{a}/_{b} \le {}^{c}/_{d}$ *then* $^{a}/_{b} \le {}^{(a+c)}/_{(b+d)} \le {}^{c}/_{d}$

for any integers a, b, c, d. He then knew that

$$3 \le \pi \le {}^{22}/_{7}$$

so, approximately,

$$\pi = 3.1415926 = {}^{(3x + 22y)}/_{(x + 7y)}$$

giving y = 16x approximately, so

$$\pi = {}^{(3x + 22 \times 16x)}/_{(x + 7 \times 16x)} = {}^{355}/_{113}.$$

This accuracy in the calculation of π was not achieved in the west for over a millennium, being discovered in 1585 by the Dutch mathematician Adrian Anthoniszoom.

Martzloff presents another possible way that Zu might have found $^{355}/_{113}$ by luck rather than mathematical skill. However, given that Zu's work was considered very difficult and advanced, it is doubtful that it was found by a lucky numerical accident.

In 656, after editing by Li Chunfeng, the treatise Zhui shu (Method of Interpolation) became a text for the Imperial examinations and it became one of The Ten Classics when reprinted in 1084. However, the Zhui shu was too advanced for the students at the Imperial Academy and it was dropped from the syllabus for that reason. This almost certainly explains why the text has not survived, being lost in the early twelfth century.

In the latter part of his life Zu Chongzhi collaborated with his son, Zu Geng (or Zu Xuan), who was also an outstanding mathematician.