U3AM at Home

Exponential Growth – The Calculation

In reference to the final paragraph of my previous article, the calculation to perform was of course 1 + 2 + 4 + 8 + 16 + … (33 terms in all). This is a geometric series to which those of you in the know could simply apply the formula $\frac{a(r^n - 1)}{r - 1}$ where a (the first term) is 1, r (the common ratio) is 2 and n (the total number of terms) is 33. The answer then simplifies to $2^{33} - 1$ which, with the aid of a calculator, comes to 8,589,934,591, a number larger than the current population of the world.

With very little mathematical knowledge the answer can also be obtained by observing the following.
The sum of the grains on the first two squares is 3 and the third square requires 4 grains to be placed on it.
The sum of the grains on the first three squares is 7 and the fourth square requires 8 grains to be placed on it.
The sum of the grains on the first four squares is 15 and yes, the fifth square does require 16 grains to be placed on it, so you may now safely assume that the sum of the grains on the first 33 squares will be one fewer than the number of grains you will need to place on the 34^{th} square.

Index notation allows us to write 2 x 2 as 2^2 , 2 x 2 x 2 as 2^3, 2 x 2 x 2 x 2 as 2^4 etc. Hence the third square requires 2^2 grains to be placed on it, the fourth square 2^3 grains, the fifth square 2^4 grains and so on. This leads to the number of grains you will need to place on the 34^{th} square being 2^{33} , and therefore to the sum of the grains on the first 33 squares being $2^{33} - 1$, the same answer as that provided by the formula.

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